Monopole Current and Unconventional Hall Response on Topological Insulator

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We study theoretically the charged current above a topological insulator (TI) separated by a ferromagnetic insulating layer. An unconventional Hall response occurs in the conducting layer on top of the TI which approaches to a constant value independent of R for $R \ll \ell$ and decays with $\propto R^{-1}$ for $R \gg \ell$, where R is the separation between TI and conducting layer and ℓ is the screening length. In the comoving frame, it can be interpreted as a monopole current attached to the TI surface. The same mechanism gives the Hall response and deflection of the electron beam injected to the surface of insulating ferromagnet. A realistic estimate of an order of magnitude shows that both effects give reasonably large signal experimentally accessible.

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Topological insulator (TI) is a new state of matter realized in the noninteracting electron systems, i.e., the nontrivial band structure characterized by the "twist" of the Bloch wavefunction in the momentum space[1][2][3][4]. As in the case of quantum Hall system, there is a gap in the bulk states, and the manifestation of the nontrivial topology appears on the surface (edge) of the three (two) dimensional TI[5]. In the case of 3D TI, there appears the helical Dirac fermions on the surface, which is robust against the disorder. This helical metal state is expected to produce the several novel properties such as the topological magneto-electric (TME) effect[5], and an image magnetic monopole when a charge is put above the TI[6]. For these effect to be observed, the time-reversal symmetry breaking is needed, which can be achieved by the ferromagnetic thin layer attached on top of TI, which induces the exchange coupling and the gap to the surface Dirac fermion and its anomalous Hall effect (AHE). Especially, when the Fermi energy lies within the gap, the Hall conductance is predicted to be quantized as $\pm e^2/(2h)$, i.e., half of the conductance unit. When this condition is satisfied, the distribution of the magnetic field outside of the TI is that given by the image magnetic monopole inside the TI. However, in realistic situation, the Fermi energy is rather difficult to control, and lies within the finite density of states of the surface Dirac fermions even with the gap opens by the exchange coupling.

When the TI surface is gapped, and the Fermi surface exactly lies in the gap, the effective electromagnetic response of a 3D TI can be described by θ-term in the Lagrangian[5],

$$\mathcal{L}_{eff} = \frac{\theta}{2\pi}\frac{\alpha}{4\pi}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} \tag{1}$$

where $F_{\mu\nu}$ is the electromagnetic field strength, and α is the fine structure constant. $\theta = 0$ for conventional insulator, while $\theta = \pm\pi$ for TI. Concerning the chiral anomaly, the sign above is decided by the direction of a magnetic field or magnetization on the TI surface. This

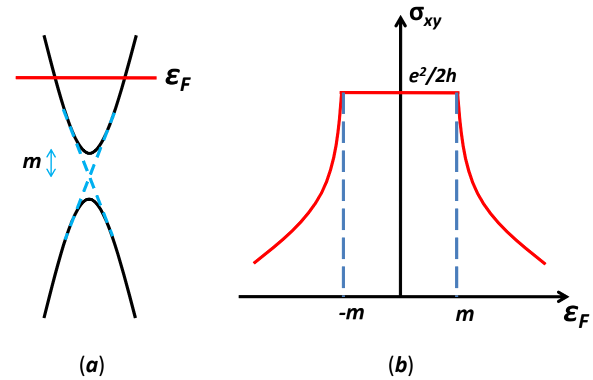

FIG. 1: Sketch of (**a**) the relative positions of Fermi surface ε_F and the magnetic gap m and (**b**) relation between Hall conductance and the Fermi surface. When the Fermi surface lies in the magnetic gap m, the conductance is quantized as half the conductance quanta. However, when the Fermi surface is pushed outside the gap, σ_{xy} decays inversely proportional to ε_F.

nonvanishing θ leads to the topological magneto-electric effect of TI. As a result, when a pure charge is placed on the top of a chirality fixed TI surface, its electric field induces a magnetic field. It's amazing that this magnetic field lines originate from the charge's mirror position with respect to the TI surface. In this sense, we may say that a charge would induce a monopole in the mirror with the help of TI[6]. Without losing the generality, assume the unity dielectric constant and magnetic permeability of the TI. The monopole strength of the induced monopole in SI units is given by $g = \frac{2\alpha\mu_0 c}{(4+\alpha^2)}q = \frac{e^2}{2h}\frac{2\mu_0}{\varepsilon_0(4+\alpha^2)}q$, with α being the fine structure constant.

Phenomenologically TME can be best interpreted as the quantum Hall effect on the TI surface by applying the bulk-edge correspondence. In the presence of a perpendicular magnetic field, quantum Hall effect with half

conductance quanta is realized for the chiral liquid on TI surface[5], namely, $\sigma_{xy} = \frac{e^2}{2h}$. Therefore, in-plane component of the electric field induced by a static charge generates a circulating Hall current. TME is nothing but the orbital magnetization due to this Hall current.

However in reality, the Fermi level does not come across the magnetic gap, but lies within in the finite density of states (Fig.1(a)). Experimentally the approachable magnetic gap is 10K at most, and it is difficult to push the Fermi surface inside the gap. On the other hand, it's quite possible that the Fermi surface lies about 100K (10meV) above or below the Dirac point.

Microscopically, the helical liquid on gapped TI surface is given by [4][5][8]:

$$H = \mathbf{k} \cdot (\sigma \times \hat{z}) + m\sigma_z \tag{2}$$

where m is the strength of perpendicular magnetization. The half-quantized Hall effect holds true only when the Fermi surface lies in the gap opened by the chirality fixing field. When the Fermi surface lies away from the gap, the conductance would not be half quantized anymore. Employing TKNN formula[10], we get the Hall conductance

$$\sigma_{xy} = \frac{m}{\varepsilon_F} \frac{e^2}{2h} \tag{3}$$

where ε_F is the Fermi surface, see Fig.1(a). It explicitly shows that the transverse conductance is suppressed by a factor of m/ε_F, so is TME and the monopole strength mentioned above. σ_{xy} for arbitrary Fermi surface is shown explicitly in Fig.1(b).

However the suppression of the Hall conductance is not the only penalty to pay. Right now the Fermi surface is intersecting the edge state, so the surface is metallic instead of the ideal insulating one mentioned above. In this case, the electric field in plane would be greatly reduced by the screening effect, leading to an additional suppression of the magneto-electric field. Due to the low and slowly-varying nature of the potential, Thomas-Fermi approximation is employed. Assume the charge density $\rho(\mathbf{r})$ on the TI surface is given by

$$\rho(\mathbf{r}) = -N_f \phi(\mathbf{r}) \tag{4}$$

where $\mathbf{r}$ is the 2D vector in plane, $\phi(\mathbf{r})$ is the scalar potential, and $N_f = e^2 E/[2\pi(v_F\hbar)^2]$ is the density of states. Here v_F is the Fermi velocity of the Dirac fermion. By the method of Green's function, we can derive the self-consistent equation in the presence of a point charge q with distance R away from the TI surface:

$$\varepsilon_0 \phi(\mathbf{r}, z) = \int d^2\mathbf{r}' \frac{\rho(\mathbf{r}')}{4\pi\sqrt{z^2 + (\mathbf{r}-\mathbf{r}')^2}} + \frac{q}{4\pi} \frac{1}{\sqrt{(z-R)^2 + \mathbf{r}^2}} \tag{5}$$

Taking the limit $z \to 0$, and applying the Fourier transformation, we finally get the polar symmetric potential distribution, given by

$$\phi(r) = \frac{q}{\varepsilon_0} \int_0^\infty \frac{dk}{(2\pi)^2} \frac{k \exp(-kR)}{2k + 1/\ell} J_0(kr) \tag{6}$$

where $J_0(x)$ is the zero-order Bessel function, and $\ell = \varepsilon/N_f$ is the screening length. Then the electric field in plane can be derived as $E(\mathbf{r}) = -\nabla\phi(\mathbf{r})$. As long as the electric field in plane is derived, the calculation of TME is straightforward. The monopole picture recovers when $\ell \to \infty$. It's worth emphasizing that although TME effect survives for finite ℓ, the monopole picture should be replaced by magnetic dipole's picture then, and it's explicitly shown in Eq.(6) that the total effect is further reduced.

As a result, this highly nontrivial TME induced by TI is unfortunately not only governed by the small number $\alpha \approx 1/137$, but also further reduced by several factors when the realistic situation is considered. So that TME is quite difficult to be observed experimentally. A way out of this embarrassing situation is to replace the original point charge by a charged current flowing in parallel with the TI surface. It appears in the following that this modification is not only quantitative, but also qualitative. Here we have to emphasize that the current and the charge are different quantities as the usual currents are neutral. Anyway, we still have certain methods to make the conducting region charged. While such charged current is available above the gapped TI surface, an in-plane electric field perpendicular to the current would be generated, see Fig.2. Consequently, orbital magnetization is induced by the Hall current J_H related to this field. Concerning the symmetry, this magnetization must be

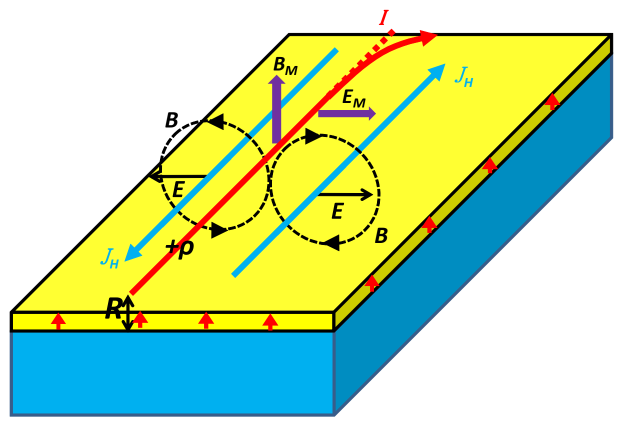

FIG. 2: (Color online) Schematic illustrations of the transverse force acting on the current. A charged current I (red line) with charge density ρ is introduced above the gapped TI surface. Its in-plane electric field E generates Hall current J_H (blue line) in parallel with the original current. Consequently, an orbital magnetization B_M (labeled by purple) is induced upward. The Lorentz force related to B_M induces an unconventional Hall response E_M transversely.

perpendicular to both the current and TI surface. Starting from Eq.(6), simple calculation shows this magnetic field is

$$B_M = \frac{\mu_0 \rho \sigma_{xy}}{8\pi\varepsilon_0 R}[1-(R/\ell)\exp(R/\ell)\Gamma(R/\ell)] \qquad (7)$$

where $\Gamma(x)$ is the Gamma function, ρ is the charge density, and R is the distance between current and TI surface. This magnetic field would naturally acts a Lorentz force on the original current, leading to a Hall response and the deflection of this charged current transversely. Quantitatively, this Lorentz force is effectively equivalent to a transverse electric field E_M:

$$E_M = \frac{\mu_0 I \sigma_{xy}}{8\pi\varepsilon_0 R}[1-(R/\ell)\exp(R/\ell)\Gamma(R/\ell)] \qquad (8)$$

This equation is the main result of this paper. The Hall response induced by this transverse electric field is a unique property of TI. In the limit $\ell \gg R$, the leading order gives $E_M = \frac{\mu_0\sigma_{xy}I}{8\pi\varepsilon_0 R}$. So the monopole's picture is recovered. While $\ell \ll R$ is small, $E_M = \frac{\mu_0\sigma_{xy}I}{8\pi\varepsilon R}\frac{\ell}{R}$, and dipole picture instead of monopole picture applies E_M is proportional to $1/R^2$ here. If R is fixed, this result shows another reduction factor of ℓ/R is required. Assume $\varepsilon_F = 10$meV, rough estimation gives $\ell \approx 500$nm for Bi_2Se_3[9]. As a result, small $\ell \ll R$ limit is adopted usually.

The previous arguments are applied in the laboratory frame. The physics behind this phenomenon can be even better understood if we check what's going on in the frame comoving with the charged current. For simplicity, assume the Fermi surface is in the magnetic gap. In the comoving frame, the charges are static while the TI surface as a whole is moving backward. By TME, magnetic monopoles exist in the mirror. However, as shown before, the physics behind these monopoles are the quantum Hall effect on the TI surface. As a consequence, these monopoles are attached to the TI surface, and are moving backward as well. Motion of the monopoles constructs a monopole current I_M in the comoving frame. To some extent, the electric-magnetic duality of Maxwell theory is completely recovered here. In analogy with the Ampere's law, this monopole current generates an electric field winding around, which provides a horizontal but transverse electric field acting on the original current. This field is exactly the effective field (Eq.(8)) derived above.

In the realistic situation, the width d of the conducting region should be considered. Generically, d is larger than the current-surface separation, as well as the screening length ℓ. Concerning this, detailed calculation shows an additional factor of R/d should be included in Eq.(8). So in the small R limit, the anomalous electric field approaches to an constant value proportional to the current density. And in the limit $\ell \ll R \ll d$, the anomalous field will be proportional to $1/R$. This result can be understood as follows. The quantity I/R in the leading term of l.h.s of Eq.(8) gives the dimension of current density. When the large width limit is considered, this quantity should be replaced by the planer current density I/d. Therefore, a factor of R/d is required. Except for a quantity close to the unity, we may actually replace I/R by $I/Max(R,d)$ for simplicity.

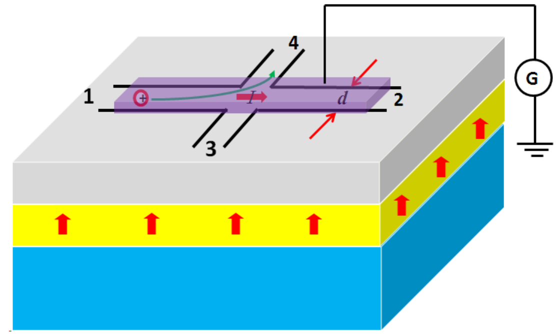

FIG. 3: Schematic illustrations of UAHE. The bottom blue layer is TI, the middle one is a magnetic layer, followed by a semiconductor layer. On the top a gate electrode is deposited. The current is flowing rightwards, and Hall voltage can be detected transversely.

Experimentally, the required charged current can be provided by the steady electron beam emitting from low-energy electron gun (LEED for example). While drifting above the TI surface, the induced anomalous electric field would significantly deflect the trajectory of the electron beam. Numerical estimation shows when the sample size is 1cm $\times$ 1cm, electron velocity is 1×10^5m/s, $m = 1$meV, $I = 1\mu$A, $d = R = 1\mu$m, the resulting transverse drift would be 5μm. These values are realistic ones for beams produced by electron guns. This deflection can be easily traced by angle resolved measurement.

In fact, the force given by the monopole current is not the only force acting on the original current. The original current generates an image current as well, which would support a Lorentz force acting on the original current itself. However this Lorentz force is actually pointing vertically, which is orthogonal to our anomalous force. As a result, these two effects can be distinguished easily.

While the original current is provided by a quantum wire deposited on the TI surface, a Hall-like effect can be observed, shown in Fig.3. The first layer on the top of TI surface is a magnetic layer with the magnetization pointing vertically. The second layer is a semiconductor layer on which quantum wire is deposited. Employ the usual four-terminal measurement, where electrodes 1 and 2 are source and drain respectively. Electric field is measured along electrodes 3 and 4. Gate electrode is deposited on the top the set up. When it is gated, we may succeed realizing a charged conducting region in

the quantum wire. In equilibrium, this measured electric field is just the transverse electric field derived in Eq.(8). Usually it is the Hall resistance R_H that is directly measured. Taking account all the factors concerned before, in the realistic case $\ell \ll R \ll d$, we have

$$R_H = \frac{h}{e^2}\alpha^2 \frac{m}{\varepsilon_F}\frac{\ell}{R} \tag{9}$$

This result shows that the Hall resistance decays inversely proportional to R, which serves as a quantitative characteristic for this monopole-induced Hall effect. If $R = 1\mu$m, and the longitude current $I = 1$mA, we have the rough estimation that $R_H \approx 0.01\Omega$, and $V_H \approx 10\mu$V. This Hall voltage can be easily measured by usual voltmeters. Actually these conditions can be further optimized. The most effective way is to drive Fermi surface closer to the Dirac point, as the Hall voltage increases with $\propto 1/\varepsilon_F^2$. Consequently this effect is quite promising to be detected in the near future.

Conventional anomalous Hall effect (AHE) in ferromagnetic metals has been well studied in the past[7]. In that case, the magnetization alone is not sufficient to support the giant Hall effect. Its role is to break the time reversal only, while it is the spin-orbit interaction that provides the driving force. The situation is similar in our setup. The ferromagnetic layer on top of TI surface only breaks the time reversal symmetry and fixes the chirality. The real driving force is the transverse electric field given by the monopole current. In this sense we name our new effect as *unconventional anomalous Hall effect*(UAHE). Actually the magnetic field provided by the magnetic layer is vanishingly small in the Hall bar measured, and the conventional Hall effect is negligible here. This magnetic field is restricted on the TI surface only.

In addition, UAHE is fundamentally different from the conventional Hall effect. For the conventional one, the Hall voltage satisfies $V_H = vBd \propto I/\rho$, where ρ is the charge density. While in the present situation, $V_H \propto \rho v \propto I$. It means that the Hall voltage here is unchanged as long as the current is fixed. On the contrary, in the conventional Hall effect, the charge density matters. If the gate changes sign, charge density and consequently Hall voltage acquire a sign change as well. From this point of view, we can easily distinguish UAHE from conventional Hall effect. Actually, even when the chirality is fixed by an external magnetic field, where UAHE coexists with conventional Hall effect, one can also separate UAHE from the conventional one effectively. In this case, we may adjust the gate voltage to vary the carrier density, and plot the Hall voltage versus inverse of the carrier density. The intercept gives the UAHE. On the other hand, we may also vary the magnetic field. The zero field limit gives the desired result as well.

It's a revealing issue when the ferromagnetic layer on TI surface is metallic, and Hall measurement is applied to this layer directly. In this case, one can have a system with coexistence of AHE and UAHE. When the layer is thin, the effective current-surface separation is small, so that anomalous electric field is large. Meanwhile, the phonon scattering greatly suppresses AHE. As a result, UAHE is overwhelming, and the net Hall conductance decreases if the layer thinkness increases. However, in the thick layer limit, UAHE is vanishingly small, and AHE is dominant. The Hall conductance would approach to a constant value. This cross-over between AHE and UAHE help us to distinguish these two effect not only conceptually, but also experimentally.

In conclusion, we have proposed an unconventional anomalous Hall effect in this work. In the laboratory frame, the upward magnetic field induced by a charged current leads to an unconventional Hall response. This effect can be explained as a monopole current in the comoving frame. This UAHE survives even when the chemical potential is away from the gap opened by chirality fixing ferromagnetic layer on top of TI. Two experiments are proposed in this paper, which hopefully provide the smoking-guns of TI.

It's also an interesting issue when the chirality of TI surface is fixed by the spin of the charge carrier itself. When the current-surface separation is quite small, the local magnetic field is provided by the local spin only. Opposite spins would lead to opposite chiralities, and the direction of local monopole current is therefore opposite. As the charge here is the same for both spin, the transverse force would be opposite. Consequently, the conventional spin Hall effect emerges and the transverse spin voltage is expected. The issue can be simplified when the incident current is spin polarized, where UAHE induced spin Hall effect would lead to a Hall voltage built between the two edges. However it should be pointed out that the gap opened by a single spin is pretty small ($T(\mathrm{K}) \propto R(\mathrm{m})^{-3}$, and $T \sim 1$K when $R = 1\mathring{A}$), and ultra-low temperature is called for.

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